Hall resistivity of granular metals

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We calculate the Hall conductivity σ_{xy} and resistivity ρ_{xy} of a granular system at large tunneling conductance $g_T \gg 1$. We show that in the absence of Coulomb interaction the Hall resistivity depends neither on the tunneling conductance nor on the intragrain disorder and is given by the classical formula $\rho_{xy} = H/(n^* ec)$, where n^* differs from the carrier density n inside the grains by a numerical coefficient determined by the shape of the grains. The Coulomb interaction gives rise to logarithmic in temperature T correction to ρ_{xy} in the range $\Gamma \lesssim T \lesssim \min(g_T E_c, E_{\mathrm{Th}})$, where Γ is the tunneling escape rate, E_c is the charging energy and E_{Th} is the Thouless energy of the grain.

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Hall resistivity (HR) of metals and semiconductors gives very important information about their properties. According to the classical Drude-Boltzmann theory HR

$$\rho_{xy} = H/(nec) \tag{1}$$

does not depend on the mean free path and allows one to experimentally determine the carrier concentration n.

Recently, much attention from both experimental and theoretical sides has been paid to granular systems (see a review [1] and references therein). Although various physical quantities have been calculated in different regimes, the Hall transport in the granular matter has not been addressed theoretically yet. In this work we calculate the Hall conductivity(HC) of a granular system and Coulomb interaction corrections to it in the metallic regime, when the intergrain tunneling conductance $G_T = (2e^2/\hbar)g_T$ is large, $g_T \gg 1$ (further we set $\hbar = 1$).

Technically, calculating HC σ_{xy} appears to be more complicated than calculating the longitudinal conductivity(LC) σ_{xx}. The granularity of the system is ensured by the condition that the conductance $G_0 = 2e^2 g_0$ of the grain is much larger than the tunneling conductance G_T, $g_0/g_T \gg 1$. In this limit the main contribution to σ_{xx} comes from the tunnel barriers between the grains rather than from scattering on impurities inside the grains. In the absence of Coulomb interaction LC equals

$$\sigma_{xx}^{(0)} = G_T a^{2-d}, \tag{2}$$

where a is the size of the grains and d is the dimensionality of the system. Therefore when studying longitudinal transport one can neglect electron dynamics inside the grains, which simplifies calculations significantly. On the contrary, for Hall transport one is forced to take the intragrain electron dynamics into account, since the Hall current originates from the transversal drift in crossed magnetic and electric fields *inside* the grains.

As we find in this work the intragrain electron dynamics can be included within the diagrammatic approach by considering higher diffusion modes inside the grain. This

procedures accounts for the finiteness of the ratio g_T/g_0, and allows one, in principle, to study both LC and HC of the granular system for arbitrary ratio g_T/g_0. The obtained results reproduce the solution of the classical electrodynamics problem for a granular medium (e.g. the formula $\sigma_{xx}^{(0)} = a^{2-d} G_T G_0/(G_T + G_0)$ can be obtained as a series in g_T/g_0). Quantum effects (e.g. Coulomb interaction and weak localization) may be incorporated into this scheme afterwards.

We perform calculations for magnetic fields H such that $\omega_H \tau \ll 1$, where $\omega_H = eH/mc$ is the Larmor frequency and τ is the electron scattering time inside the grain. Since the effective mean free path $l = v_F \tau \lesssim a$ does not exceed the grain size a, and typically $a \approx 10 - 100$nm, the condition $\omega_H \tau \ll 1$ is well fulfilled for all experimentally available fields H.

Let us list the main results of this work. First we neglect Coulomb interaction completely and obtain for HC in the lowest nonvanishing order in g_T/g_0

$$\sigma_{xy}^{(0)} = G_T^2 R_H a^{2-d}, \tag{3}$$

where R_H is the classical Hall resistance of a single grain (see Fig. 1(right)). This result obtained by diagrammatic methods is completely classical provided the tunneling contact is viewed as a surface resistor with conductance G_T. The HR of the system

$$\rho_{xy}^{(0)} = R_H a^{d-2} = H/(n_d^* ec) \tag{4}$$

is given by the Hall resistance of a single grain R_H, which depends on *the geometry of the grain* but *not on the intragrain disorder*. Eq. (4) defines the effective carrier density n_d^* of the granular medium. For a three-dimensional ($d = 3$, many granular monolayers) array $n_3^* = An$ differs from the electron density n in the grain by a numerical factor A, $0 < A \leq 1$, determined by the grain geometry [14]. For grains of a simple geometry this factor is given by the ratio of the largest cross section area S to the cross section area of the lattice cell a^2: $A = S/a^2$. So, $A = 1$ for cubic grains ($S = a^2$), $A = \pi/4$ for spherical grains

$(S = \pi a^2/4)$. For a two-dimensional ($d = 2$, single granular monolayer) array the 3D result must be multiplied by the thickness of the layer a: $n_2^* = aAn$.

Next we calculated the first-order correction to HC $\sigma_{xy}^{(0)}$ (Eq. (3)) due to Coulomb interaction at temperatures $T \gtrsim \Gamma$ not much smaller than the tunneling escape rate $\Gamma = g_T \delta$ (δ is the mean level spacing of the grain). We find significant T-dependent corrections in the range $\Gamma \lesssim T \lesssim g_T E_c$ ($E_c = e^2/a$ is the charging energy of the grain), whereas for $T \gtrsim g_T E_c$ the relative corrections are of the order of $1/g_T$ or smaller [15].

There exist two different contributions to HC

$$\sigma_{xy} = \sigma_{xy}^{(0)} + \delta\sigma_{xy}^{(1)} + \delta\sigma_{xy}^{(2)}. \tag{5}$$

One of them, $\delta\sigma_{xy}^{(1)}$, can be attributed to the renormalization of individual tunneling conductances G_T (*tunneling anomaly* [2, 3, 4]) in a granular medium and has the form:

$$\frac{\delta\sigma_{xy}^{(1)}}{\sigma_{xy}^{(0)}} = -\frac{1}{\pi g_T d} \ln\frac{g_T E_c}{T}. \tag{6}$$

The other one, $\delta\sigma_{xy}^{(2)}$, is due to *elastic cotunneling* (EC)[5], it involves diffusion of an electron through the grain (that is why it is suppressed at temperatures greater than the Thouless energy of the grain E_{Th}):

$$\frac{\delta\sigma_{xy}^{(2)}}{\sigma_{xy}^{(0)}} = \frac{c_d}{4\pi g_T} \ln\left[\frac{\min(g_T E_c, E_{\mathrm{Th}})}{T}\right]. \tag{7}$$

where c_d is a numerical factor Eq. (16). Since the correction $\delta\sigma_{xy}^{(1)}$ merely renormalizes the conductance G_T in Eq. (3), it does not affect the HR $\rho_{xy} = \sigma_{xy}/\sigma_{xx}^2$. Indeed the correction Eq. (6) is cancelled by the corresponding correction to σ_{xx} [1, 6, 7], describing renormalization of G_T in Eq. (2). Therefore the total correction $\delta\rho_{xy}$ to HR $\rho_{xy} = \rho_{xy}^{(0)} + \delta\rho_{xy}$ is due to the EC effect (Eq. (7)) only:

$$\frac{\delta\rho_{xy}}{\rho_{xy}^{(0)}} = \frac{\delta\sigma_{xy}^{(2)}}{\sigma_{xy}^{(0)}} = \frac{c_d}{4\pi g_T} \ln\left[\frac{\min(g_T E_c, E_{\mathrm{Th}})}{T}\right]. \tag{8}$$

We conclude that the Hall resistivity ρ_{xy} of a granular metal at temperatures $T \gtrsim \min(g_T E_c, E_{\mathrm{Th}})$ is given by Eq. (4) and is independent of the intragrain and tunnel contact disorder. Measuring ρ_{xy} at such T and using Eq. (4) one can extract an important characteristic of the granular system: its effective carrier density n_d^*. At temperatures $\Gamma \lesssim T \lesssim \min(g_T E_c, E_{\mathrm{Th}})$ Coulomb interaction leads to $\ln T$-dependent corrections to ρ_{xy}. Comparison of Eqs. (4)-(8) with experimental data may serve as a good check of the theory developed here. The logarithmic dependence $\rho_{xx} = a + b\ln T$ of granular metals has been observed experimentally [8], and ρ_{xy} can also be measured [9]. Our theory may also be applied to indium tin oxide(ITO) materials (see e.g. [10]). Another related
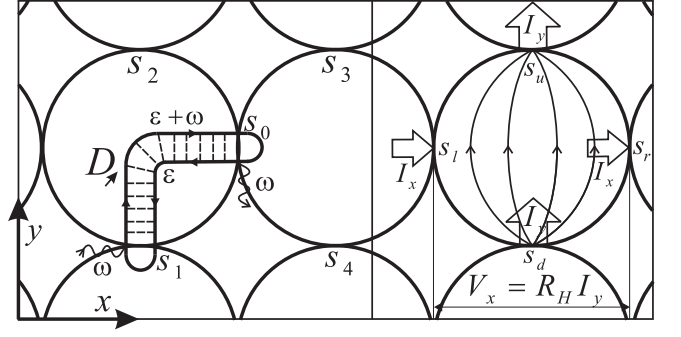

FIG. 1: (left) Diagrams for the Hall conductivity $\sigma_{xy}^{(0)}$ of the granular system (Eqs. (14)). The diffuson $D_{\nearrow}$ connecting contact $\mathbf{s}_1$ to $\mathbf{s}_0$ is shown. External tunneling vertices (wavy lines) must be attached in 4 possible ways. Three more diffusons $D_{\searrow}, D_{\swarrow}, D_{\nwarrow}$ connecting contacts $\mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$ to $\mathbf{s}_0$, respectively, must be also taken into account. (right) Classical picture of Hall conductivity of a granular system. The current $I_y = G_T V_y$ running through the grain in y-direction causes the Hall voltage drop $V_H = R_H I_y$ between its opposite banks in x-direction, which is also applied to the contacts (the total voltage drop per lattice period in x-direction is 0) giving the Hall current $I_x = G_T V_H = G_T^2 R_H V_y$ (Eq. (3)).

effect is the anomalous Hall effect in ferromagnetic granular materials [11].

Now we briefly outline the model and method used to derive the announced results, details of our calculations will be presented elsewhere[12]. We consider a quadratic($d = 2$) or cubic($d = 3$) lattice of identical (in form and size) grains coupled to each other by tunnel contacts (Fig. 1). To simplify the calculations we assume the intragrain electron dynamics diffusive, $l \ll a$. However, our results are also valid for ballistic ($l \sim a$) intragrain disorder. In the metallic regime ($g_T \gg 1$) the Coulomb interaction effects can be considered as a perturbation.

We write the Hamiltonian describing the system as

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_c.$$

$$\hat{H}_0 = \sum_{\mathbf{i}} \int d\mathbf{r_i} \psi^\dagger(\mathbf{r_i}) \left[\xi\left(\mathbf{p_i} - e/c\mathbf{A}(\mathbf{r_i})\right) + U(\mathbf{r_i})\right] \psi(\mathbf{r_i})$$

is the electron Hamiltonian of isolated grains, $\xi(\mathbf{p}) = \mathbf{p}^2/2m - \epsilon_F$, $\mathbf{A}(\mathbf{r_i})$ is the vector potential describing uniform magnetic field $\mathbf{H} = H\mathbf{e}_z$, $U(\mathbf{r_i})$ is the random disorder potential of the grains, $\mathbf{i} = (i_1, \ldots, i_d)$ is an integer vector numerating the grains. The disorder average is performed using a Gaussian distribution with the variance $\langle U(\mathbf{r_i})U(\mathbf{r'_i})\rangle = \frac{1}{2\pi\nu\tau}\delta(\mathbf{r_i} - \mathbf{r'_i})$, where ν is the density of states in the grain at the Fermi level per one spin direction. Further, the tunneling Hamiltonian is given by

$$\hat{H}_t = \sum_{\langle \mathbf{i},\mathbf{j}\rangle} X_{\mathbf{ij}} = \sum_{\langle \mathbf{i},\mathbf{j}\rangle} \int d\mathbf{s_i}d\mathbf{s_j}\, t(\mathbf{s_i}, \mathbf{s_j})\psi^\dagger(\mathbf{s_i})\psi(\mathbf{s_j})$$

the summation is taken over the neighboring grains connected by a tunnel contact, and the integration is done over the surfaces of the contact. The inevitable irregularities of the tunnel contact are modelled by the relation $\langle t(\mathbf{s_i}, \mathbf{s_j}) t(\mathbf{s_j}, \mathbf{s_i}) \rangle = t_0^2 \delta(\mathbf{s_i} - \mathbf{s_j})$, where $\delta(\mathbf{s_i} - \mathbf{s_j})$ is a δ-function on the contact surface, t_0^2 has the meaning of tunneling probability per unit area of the contact. The Coulomb interaction can be taken in the form (e.g. [13])

$$\hat{H}_c = \frac{e^2}{2} \sum_{\mathbf{i},\mathbf{j}} \frac{n_\mathbf{i} n_\mathbf{j}}{C_{\mathbf{ij}}},$$

where $n_\mathbf{i} = \int d\mathbf{r_i}\, \psi^\dagger(\mathbf{r_i})\psi(\mathbf{r_i}) - \bar{n}$ is the excess number of electrons in the $\mathbf{i}$-th grain, $C_{\mathbf{ij}}$ is the capacitance matrix.

The conductivity is calculated using the Kubo formula:

$$\sigma_{\mathbf{ab}}(\omega) = 2e^2 a^{2-d} \frac{1}{\omega} \sum_\mathbf{j} (\Pi_{\mathbf{ab}}(\omega, \mathbf{i}-\mathbf{j}) - \Pi_{\mathbf{ab}}(0, \mathbf{i}-\mathbf{j})) \quad (9)$$

where $\omega = 2\pi T k > 0$ is a bosonic Matsubara frequency, $\mathbf{a}$ and $\mathbf{b}$ are the lattice unit vectors,

$$\Pi_{\mathbf{ab}}(\omega, \mathbf{i}-\mathbf{j}) = \int_0^{1/T} d\tau\, e^{i\omega\tau} \langle T_\tau I_{\mathbf{i},\mathbf{a}}(\tau) I_{\mathbf{j},\mathbf{b}}(0) \rangle \quad (10)$$

is the current-current correlator, $I_{\mathbf{i},\mathbf{a}}(\tau) = X_{\mathbf{i}+\mathbf{a},\mathbf{i}}(\tau) - X_{\mathbf{i},\mathbf{i}+\mathbf{a}}(\tau)$, the thermodynamic average $\langle \ldots \rangle$ is taken with $\hat{H}$, $A(\tau) = e^{\hat{H}\tau} A e^{-\hat{H}\tau}$ is the Heisenberg operator.

We perform calculations of $\sigma_{\mathbf{ab}}$ using diagrammatic technique. Let us first neglect Coulomb interaction $\hat{H}_c$. Technically, one expands Eq. (10) both in disorder potential $U(\mathbf{r_i})$ and the tunnelling Hamiltonian $\hat{H}_t$ and averages over the intragrain and contact disorder.

Summing the diagrams corresponding to intragrain motion we obtain an important object: the diffusion propagator (*diffuson*) of a single isolated grain given by a disorder-averaged product of two Green functions:

$$D(\omega, \mathbf{r}, \mathbf{r}') = \frac{1}{2\pi\nu} \langle G(\varepsilon+\omega, \mathbf{r}, \mathbf{r}') G(\varepsilon, \mathbf{r}', \mathbf{r}) \rangle, \quad (\varepsilon+\omega)\varepsilon < 0.$$

In the presence of magnetic field ($\omega_H \tau \ll 1$) this propagator satisfies the equation

$$(|\omega| - D_0 \nabla_\mathbf{r}^2) D(\omega, \mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \quad (11)$$

and the boundary condition at the grain surface

$$(\mathbf{n}, \nabla_\mathbf{r} D)|_{\mathbf{r}\in S} = \omega_H \tau (\mathbf{t}, \nabla_\mathbf{r} D)|_{\mathbf{r}\in S}. \quad (12)$$

Here $D_0 = v_F l/3$ is the diffusion coefficient in the grain, $\mathbf{n}$ is the normal unit vector pointing outside the grain, $\mathbf{t} = [\mathbf{H}, \mathbf{n}]/|[\mathbf{H}, \mathbf{n}]|$ is the tangent unit vector pointing in the direction of the edge drift. Eq. (12) is due to the fact that the current component normal to the grain surface vanishes, the right hand side describes the edge

drift caused by magnetic part of the Lorentz force. The solution to Eqs. (11),(12) can be presented in the form

$$D(\omega, \mathbf{r}, \mathbf{r}') = \frac{1}{|\omega|V} + \sum_{n>0} \frac{\phi_n(\mathbf{r})\phi_n^*(\mathbf{r}')}{|\omega| + \gamma_n} \quad (13)$$

where ϕ_n are the eigenfunctions of the problem

$$-D_0 \nabla_\mathbf{r}^2 \phi_n = \gamma_n \phi_n, \quad (\mathbf{n}, \nabla_\mathbf{r}\phi_n)|_S = \omega_H \tau (\mathbf{t}, \nabla_\mathbf{r}\phi_n)|_S.$$

There always exists a uniform solution $\phi_0(\mathbf{r}) = 1/\sqrt{V}$ (V is the grain volume) with zero eigenvalue $\gamma_0 = 0$ giving the 0-th harmonic $1/|\omega|V$ in Eq. (13).

Accounting for tunneling processes makes the transport between different grains possible. The diagrams for Eq. (10) in the $2N$-th order in $\hat{H}_t$ are given by diffusons connecting the contacts $(\mathbf{i}+\mathbf{a}, \mathbf{i})$ and $(\mathbf{j}+\mathbf{b}, \mathbf{j})$ that run through the other $N - 1$ contacts in all possible ways. An important observation is that the diffusons inside the grain always enter the expression for $\sigma_{\mathbf{ab}}$ as a *difference* $D(\omega, \mathbf{s}_1, \mathbf{s}_2) - D(\omega, \mathbf{s}_3, \mathbf{s}_4)$ of the diffusons connecting different contacts. Therefore the 0-th harmonic drops out and the contribution to $\sigma_{\mathbf{ab}}$ comes from nonzero harmonics, each 2 powers of $\hat{H}_t$ bringing a factor $\Gamma/E_{\text{Th}} = g_T/g_0$.

For the longitudinal conductivity ($\mathbf{a} = \mathbf{b} = \mathbf{e}_x$) the above procedure would lead to a small contribution in the limit $g_T/g_0 \ll 1$, since Eq. (2) can be obtained from a *single contact* ($\mathbf{j} = \mathbf{i}$) without expanding Eq. (10) in $\hat{H}_t$ ($N = 0$). For the Hall conductivity ($\mathbf{a} = \mathbf{e}_x, \mathbf{b} = \mathbf{e}_y$) considering the higher diffusion modes is inevitable, because even in the lowest nonvanishing order in g_T/g_0 we have to connect *different contacts* by diffusons of a single grain as shown in Fig. 1(left) ($\mathbf{j} = \mathbf{i}, \mathbf{i}+\mathbf{e}_x, \mathbf{i}-\mathbf{e}_y, \mathbf{i}+\mathbf{e}_x - \mathbf{e}_y$). Doing so, we obtain

$$\sigma_{xy}^{(0)}(\omega) = 2e^2 a^{2-d} \frac{g_T^2}{\nu} (D_\nearrow - D_\searrow + D_\swarrow - D_\nwarrow), \quad (14)$$

where $g_T = 2\pi(\nu t_0)^2 \Sigma$ is the conductance of a tunnel contact, Σ is the area of the contact, $D_\alpha = \frac{1}{\Sigma^2} \int d\mathbf{s}_0 d\mathbf{s}_a \bar{D}(\mathbf{s}_0, \mathbf{s}_a)$, $a = 1, 2, 3, 4$ for $\alpha = \nearrow, \searrow, \swarrow, \nwarrow$ respectively (Fig. 1(left)). Here

$$\bar{D}(\mathbf{r}, \mathbf{r}') = \sum_{n>0} \phi_n(\mathbf{r})\phi_n^*(\mathbf{r}')/\gamma_n$$

is the diffuson without the 0-th harmonic at $\omega = 0$ satisfying Eqs. (11),(12) with $\omega = 0$ [16]. Retaining only the 0-th harmonic in Eq. (13) would give just 0 in Eq. (14) and we are forced to take all higher harmonics into account. Eq. (14) is nonzero for $H \neq 0$ since the edge trajectories for $D_\nearrow = D_\swarrow$ are shorter (if $e > 0$ is assumed) than for $D_\searrow = D_\nwarrow$, and therefore $D_\nearrow - D_\searrow = D_\swarrow - D_\nwarrow > 0$.

In fact, the result Eq. (14) is purely classical, provided one treats the tunnel contact as a surface resistor with the conductance G_T. Indeed, the classical HC of the granular medium can be easily presented in the form of

Eq. (3) (Fig. 2(right)). The Hall resistance R_H of the grain is defined via the difference (Hall voltage) of electric potential $\varphi(\mathbf{r})$ between the opposite banks of the grain, $V_H = \varphi(\mathbf{s}_r) - \varphi(\mathbf{s}_l) = R_H I_y$, when the current $I_y = I$ passes through the grain. The current density $\mathbf{j}(\mathbf{r}) = -\hat{\sigma}_0 \nabla \varphi(\mathbf{r})$ ($\hat{\sigma}_0$ is the conductivity tensor) satisfies the continuity equation $\mathrm{div}\mathbf{j} = q(\mathbf{r})$ and the boundary condition $(\mathbf{n} \cdot \mathbf{j})|_S = 0$. The charge source function $q(\mathbf{r})$ is nonzero on the contacts surface only and $\int d\mathbf{s}_d q(\mathbf{s}_d) = -\int d\mathbf{s}_u q(\mathbf{s}_u) = I$. Therefore $\varphi(\mathbf{r})$ satisfies

$$-\nabla_{\mathbf{r}}^2 \varphi = q(\mathbf{r})/\sigma_{xx}^{\mathrm{gr}}, \quad (\mathbf{n}, \nabla\varphi)|_S = \omega_H \tau(\mathbf{t}, \nabla\varphi)|_S. \quad (15)$$

Comparing Eq. (15) with Eqs. (11),(12) we see that $\bar{D}(\mathbf{r}, \mathbf{r}')$ is a Green function for the problem Eq. (15). Thus the solution to Eq. (15) is

$$\varphi(\mathbf{r}) = \frac{1}{\nu} \frac{I}{\Sigma} \left(\int d\mathbf{s}_d \bar{D}(\mathbf{r}, \mathbf{s}_d) - \int d\mathbf{s}_u \bar{D}(\mathbf{r}, \mathbf{s}_u) \right)$$

and Eq. (14) leads to Eq. (3) (Einstein relation $\sigma_{xx}^{\mathrm{gr}} = 2e^2 \nu D_0$ was used). This establishes the connection between our diagrammatic approach of considering higher diffusion modes and the solution of the classical electrodynamics problem for the granular system.

Luckily, for simple geometries (cubic, spherical) of the grain the Hall resistance R_H can be obtained from symmetry arguments without solving the problem Eq. (15). In such cases the Hall voltage equals $V_H = \rho_{xy}^{\mathrm{gr}} aI/S$, where S is the area of the largest cross section of the grain, $\rho_{xy}^{\mathrm{gr}} = H/(nec)$ is the specific HR of the grain material expressed in terms of the carrier density n inside the grain. Therefore, $R_H = \rho_{xy}^{\mathrm{gr}} a/S$ and the HR of the granular medium can be expressed in the form of Eq. (4), where $n_d^* = a^{d-3} An$, $A = S/a^2 \leq 1$. The quantity n_d^* defines the effective carrier density of the system.

Diagrammatic approach allows us to incorporate quantum effects of Coulomb interaction on Hall conductivity into the developed scheme. We omit details here leaving them for a more comprehensive version [12]. We consider the range of not very low temperatures $T \gtrsim \Gamma$ where we can neglect the large scale contributions analogous to those for homogeneously disordered metals [2]. We find that in this regime in the first order in the screened Coulomb interaction two contributions exist ($i = 1, 2$):

$$\delta\sigma_{xy}^{(i)}(\omega) = 2e^2 a^{2-d} \frac{g_T^2}{\nu} \sum_{n>0} (f_{n,\nearrow} - f_{n,\searrow} + f_{n,\swarrow} - f_{n,\nwarrow}) \lambda_n^{(i)}(\omega)$$

where $f_{n,\alpha} = \frac{1}{\Sigma^2} \int d\mathbf{s}_0 d\mathbf{s}_a \phi_n(\mathbf{s}_0) \phi_n^*(\mathbf{s}_a)$, $a = 1, 2, 3, 4$ for $\alpha = \nearrow, \searrow, \swarrow, \nwarrow$, respectively (Fig. 1(left)), and

$$\lambda_n^{(1)}(\omega) = -2(2\pi) T^2 \sum_{\Omega}{}' 4\frac{V_0(\Omega) - V_1(\Omega)}{\gamma_n \Omega^2},$$

$$\lambda_n^{(2)}(\omega) = 2(2\pi) T^2 \sum_{\Omega}{}' \frac{V_0(\Omega) + V_2(\Omega) - 2V_1(\Omega)}{(\omega + \Omega + \gamma_n)\Omega^2}.$$

Here $2\pi T \sum_{\Omega}{}' F(\Omega) = \sum_{0 < \Omega \leq \omega} \Omega F(\Omega) + \sum_{\omega < \Omega} \omega F(\Omega)$, $V_0 = V(\mathbf{i}, \mathbf{i})$, $V_1 = V(\mathbf{i} + \mathbf{e}_x, \mathbf{i})$, $V_2 = V(\mathbf{i} + \mathbf{e}_x + \mathbf{e}_y, \mathbf{i})$ are the components of the screened Coulomb interaction $V(\Omega, \mathbf{i}, \mathbf{j}) = \int \frac{a^d d^d \mathbf{q}}{(2\pi)^d} e^{ia\mathbf{q}(\mathbf{i}-\mathbf{j})} V(\Omega, \mathbf{q})$ with

$$V(\Omega, \mathbf{q}) = E_c(\mathbf{q})/[1 + 2E_c(\mathbf{q})\Gamma_{\mathbf{q}}/(\delta|\Omega|)],$$

$\Gamma_{\mathbf{q}} = 2\Gamma \sum_{\beta} (1 - \cos q_{\beta} a)$, $E_c(\mathbf{q}) = \sum_{\mathbf{i}} e^{-ia\mathbf{q}(\mathbf{i}-\mathbf{j})} e^2/C_{\mathbf{i}-\mathbf{j}}$ ($\beta = x, y$ for $d = 2$ and $\beta = x, y, z$ for $d = 3$, $\Omega = 2\pi Tm$, $m \in \mathbb{Z}$). The contribution $\delta\sigma_{xy}^{(1)}$ renormalizes individual tunneling conductances G_T in Eq. (3), whereas the contribution $\delta\sigma_{xy}^{(2)}$ is due to elastic cotunneling (note the diffuson $1/(\omega + \Omega + \gamma_n)$ of the grain in the expression for $\lambda_n^{(2)}$). The large logarithms in $\lambda_n^{(1)}(\omega)$ and $\lambda_n^{(2)}(\omega)$ come from frequencies $\Omega \lesssim g_T E_c$, for which the Coulomb potential is completely screened: $V(\Omega, \mathbf{q}) = \delta|\Omega|/2\Gamma_{\mathbf{q}}$. Performing analytical continuation and extracting $\sigma_{xy}^{(0)}$ with the help of Eq. (14) we arrive at Eqs. (6),(7) with

$$c_d = \int \frac{a^d d^d \mathbf{q}}{(2\pi)^d} \frac{(1 - \cos q_x a)(1 - \cos q_y a)}{\sum_{\beta}(1 - \cos q_{\beta} a)}. \quad (16)$$

In conclusion, we presented a theory of the Hall conductivity of granular metals. We have shown that at high enough temperatures the Hall resistivity is given by a classical expression, from which one can extract the effective carrier density of the system. At lower temperatures charging effects give a logaritmic temperature dependence of Hall resistivity. We hope that our predictions may be rather easily checked experimentally.

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[14] Note that although the granular array may be two- ($d = 2$) or three-dimensional ($d = 3$), the grains themselves are three-dimensional, and n is a three-dimensional density.

[15] We do not consider weak localization effects in this paper assuming they are suppressed by either magnetic field or inelastic processes.

[16] To get a correct ω-dependence of Eq. (14) (its *independence* of ω) one should renormalize the diffusons by the Coulomb interaction inside the grain. This results in the replacement $1/(|\omega| + \gamma_n) \rightarrow 1/\gamma_n$ [12].